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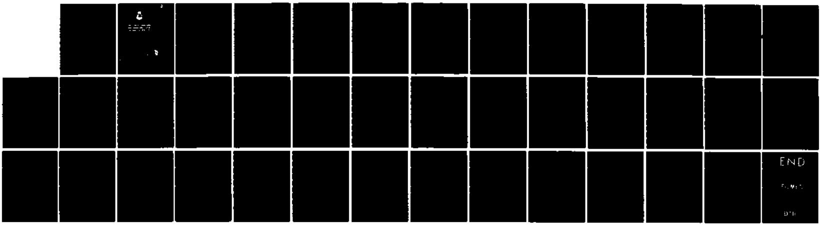

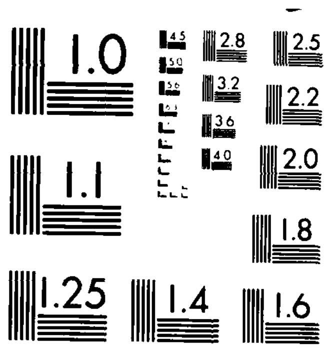

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SIMULATION OF A SPACE BASED RADAR SURVEILLANCE SYSTEM I: MATHEMATICAL ASPECTS (U)

by

N. Brousseau

DEFENCE RESEARCH ESTABLISHMENT OTTAWA
TECHNICAL NOTE 83-6

Canada

March 1983
Ottawa

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ABSTRACT

The mathematical studies necessary to implement a Space Based Radar Surveillance system simulation are presented. The movements of the satellites, the coverage area of the radars, the scanning process, the movement of the airplanes, the detection of the presence of airplanes and the calculations of the matrix used to rotate the system of coordinates are described.

RÉSUMÉ

On présente les traitements mathématiques utilisés pour réaliser la simulation d'un système de surveillance par radars montés sur satellites. On y décrit les mouvements des satellites, les zones de couverture radar, les procédures de balayages, les mouvements des avions ainsi que la détection de leur présence et le calcul des matrices effectuant des rotations de coordonneés.

ACKNOWLEDGEMENT

The author would like to thank Mr. R. Martin, Mr. B. Rook, Dr. B. Bridgewater and Dr. B. Young for many helpful discussions.

The particular approach that we used to handle the data related to the movement of the airplanes was a suggestion of Dr. B. Bridgewater and is thankfully acknowledged.

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INTRODUCTION

The purpose of this document is to present the mathematical treatments used in the description of the various aspects of a computer simulation of a Space Based Radar Surveillance (SBRS) system, bearing in mind that the validity of the results of a simulation rests on the conditions of validity of the mathematical models describing the various phenomena involved. We will therefore make a special effort to establish as clearly as possible the conditions of validity of the equations and the limitation of the various models that we used.

Our objective in making a computer simulation of a SBRS system is to be able to simulate, as accurately as possible, the interaction of a SBRS system with various attack scenarios. The duration of these attacks is limited to a few hours. From that consideration comes our first assumption: we will neglect the movement of the Earth around the Sun. The Earth will be considered as having a fixed center of rotation.

MOVEMENT OF THE SATELLITES

The motion of the satellite is assumed to obey the unperturbed Kepler laws. It means that the Earth is represented by a homogenous sphere with its mass concentrated at the center. The orbit of the satellites are then perfect ellipses. No perturbations of any kind are included in the calculations of the satellite position. The true anomaly f is then a function of E, the eccentric anomaly (1):

$$f = \sin^{-1}\left(\frac{\sqrt{1-e^2}\,\sin E}{1-e\cos E}\right) \tag{1}$$

But the eccentric anomaly E is a transcendental equation of time:

$$E - e\sin E = \left(\frac{\mu}{\ell^3}\right)^{1/2}(t - T) \tag{2}$$

where: T = integration constant determined by specifying the time origin.

μ = $k^2 m$
k^2 = gravitational constant
m = mass of the Earth
ℓ = semilatus rectum
e = eccentricity

We solve the equation 2 by using the Brown's method as described in reference 1. The true anomaly was then calculated from equation 1. The radial distance r is given by

$$r = \frac{\ell}{1 + \cos f} \tag{3}$$

The movements of the satellite in the orbital plane is then fully described by the polar coordinates (r, f) as illustrated in Figure 1.

We set the time origin such that for T = 0 the satellite is at its perigee. We also set a system of rectangular coordinate ERE whose origin is at the center of the Earth. The X axis is pointing toward the perigee and the Z axis is perpendicular to the orbital plane. The satellite turns in the positive direction. The direction of rotation of the satellite may be inverted by inverting the direction of the Z axis (see Figure 2). The orientation of the X,Y and Z axis of the ERF system is independant of time since the rotation of the Earth around the Sun is neglected.

It is also convenient to define two other systems of coordinats having their origin at the center of the Earth. The first system is called the ERF system. Its Z axis is pointing toward the North Pole and its X axis is in the equatorial plane of the Earth at a longitude of 0° at and only at time T = 0. The orientation of the X,Y and Z axis of the ERF system is not a function of time (see Fig. 3).

The second system, the ERR system, is similar to the ERF system except that the X' axis is always pointing at longitude 0, in such a way that the system is rotating with the angular speed of the Earth. The XYZ and X'Y'Z' systems are illustrated in Figure 3. The geographical coordinates (EGR) of latitude and longitude are illustrated in Figure 4.

A third system, the ORR system is used to display the results on a Tektronix screen. It is a system that is topocentric to some point of observation defined by the operator. That point of observation is rotating with the Earth. The origin of the ORR system is at the point of observation. The X axis is pointing away from the center of the Earth, the Z axis is pointing northward and the Y axis is always in a plane parallel to the equatorial plane (see Fig 5). Because of the definition of the system of coordinates the Y and Z components only are displayed on the screen.

In order to find the position of the satellite at a certain time in the ORR system, the position of the satellite is first calculated in the ERE systems, then those coordinates are transfered to the ERF system, and finally to the ORR system, taking into account the Earth rotation when necessary. The transfers are implemented using a matrix of rotation calculated by the routine ANGROT and proper translations.

A set of parameters that describes the orbit of the satellite without redundancy or ambiguity in a way that is independant of time has been selected. Those parameters are stored in an array called SYSAT whose format is described in Figure 6. The direction cosine are calculated in the ERF system. The user of the simulation can change the content of SYSAT at will.

COVERAGE AREAS

We build in our simulation the two types of coverage areas associated with the electronic scanning mode (see Fig. 7) and with the mechanical scanning mode (see Fig. 8). The blind spot centered at nadir has been set at a grazing angle of 60°. The outer limit of the area is defined in the eleventh column of SYSAT. The width of the mechanically scanned area is either set in SYSAT (workspace CURAR) or calculated from the radar parameters (workspace SCAN).

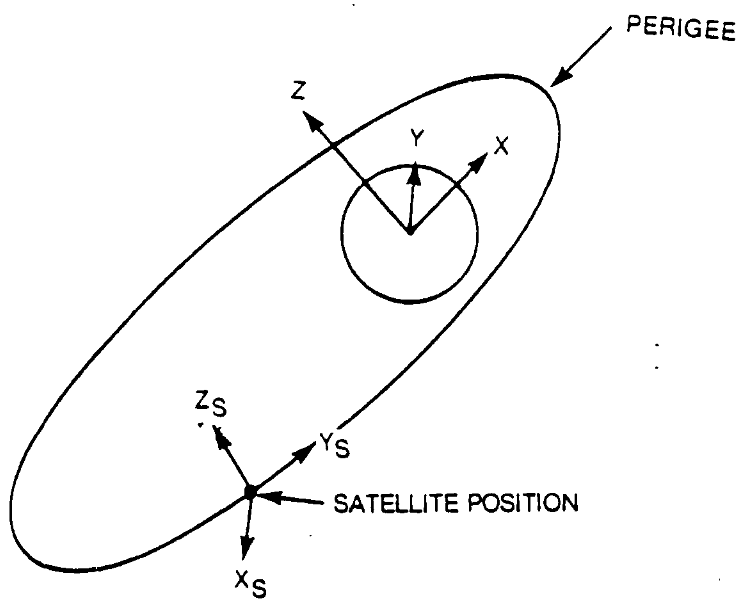

FIGURE 2 - THE ORBITAL PLANE SYSTEM OF COORDINATES (ERE)

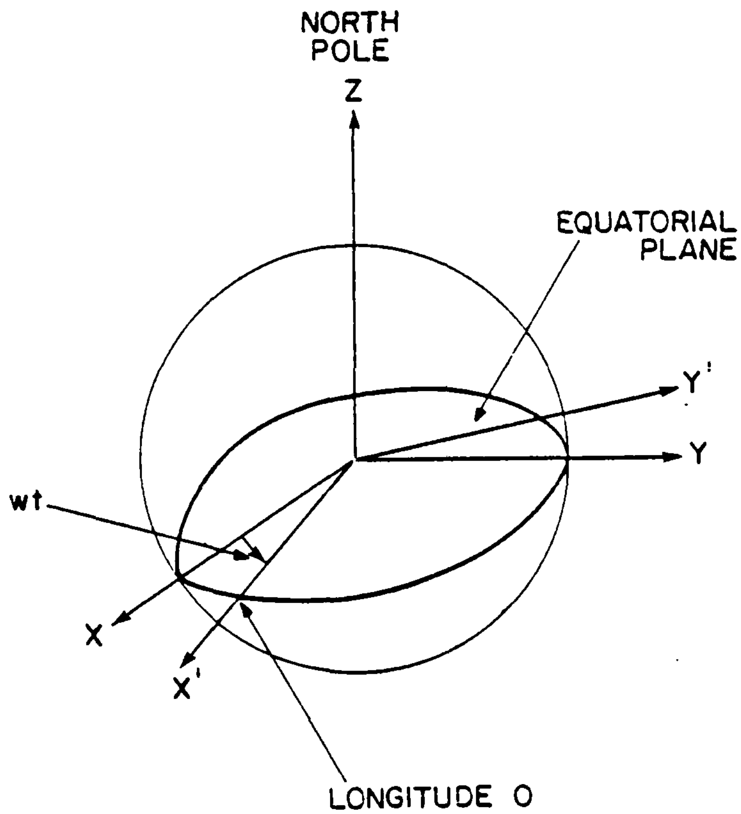

FIGURE 3 - RELATIONSHIP BETWEEN THE FIXED AND ROTATING SYSTEM OF COORDINATES (ERF AND ERR SYSTEMS)

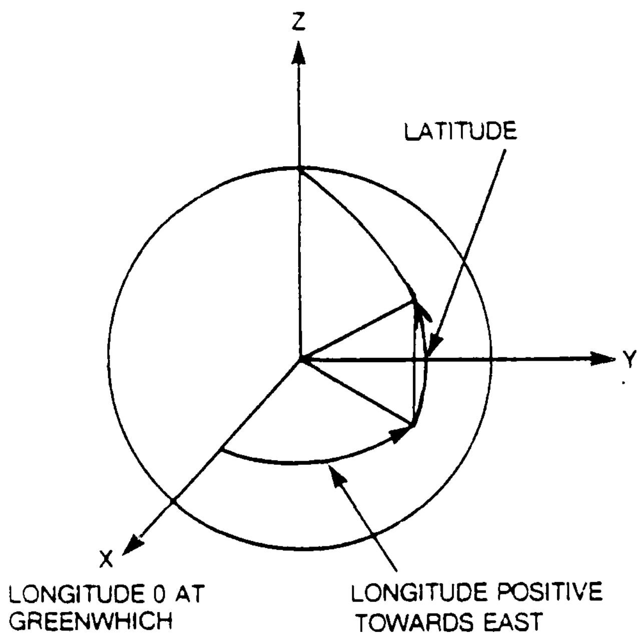

FIGURE 4 - GEOGRAPHICAL SYSTEM OF COORDINATES (EGR)

The size of the footprint on the ground is defined by the angular width of the beam and the geometry of the system as illustrated in Figure 16. The relationship between the parameters are given by the equations 4,5,6, 7,8 and 9 applied respectively to the far and the near border of the footprint. The user has to choose the amount of overlap of successive foot prints. A few examples are shown on Figure 17. The distance between the center of the footprints is given by

$$D = \left(\frac{100 - \Theta}{50}\right) \times R.$$

where Θ is the amount of overlap in %.

We have assumed here that two successive footprints have the same size and that the curvature of the Earth is negligible when we consider two successive footprints.

MOVEMENT OF AIRPLANES

The movement of the airplanes are defined and controlled by the data stored in the array FP whose format is illustrated in Figure 18. To every airplane corresponds one plane of information in FP. The first line of each plane contains the initial parameters of the airplane:

- altitude, longitude and latitude of the initial position of the airplane
- the initial horizontal speed, heading and climb rate of the airplane.
- the current time that is reset to 0 before every run of the simulation
- the start time of the airplane.

The second and third line contains the data necessary to define a change of flight plan at a certain time breakpoint. They contain

- the new horizontal speed, heading and climb rate,
- the time of the breakpoint

The rest of the line does not have to be predefined by the operator it is used to store the data during the calculations (see Figure 19).

The fourth line contains the termination time of the flight plane (that is defined by the operator) and will contain, after the execution of the program, the final position of the airplane.

When the program is run, it reads the flight plan stored in FP, calculates the position of the airplane at the required time accordingly to the flight plan and stores the current time and the position of the airplane in FP. If the current time is between the start time and the time of breakpoint 1, this is done in line two of the appropriate plane; if the current time is between the time of breakpoint 1 and the time of breakpoint 2, the storage is done in line 3, etc... So, at the end of the execution, the final position of the airplane is stored in line 4 (see Figure 20).

(METER) RADAR WAVELENGTH	(METER) ALONG-TRAKS ANTENNA SIZE	(METER) ACROSS-TRACKS ANTENNA SIZE	(METER) ALONG TRACKS OVERLAP	(METER) ACROSS-TRACKS OVERLAP	NUMBER OF REPEAT

FIGURE 15 - FORMAT OF THE RADPAR ARRAY

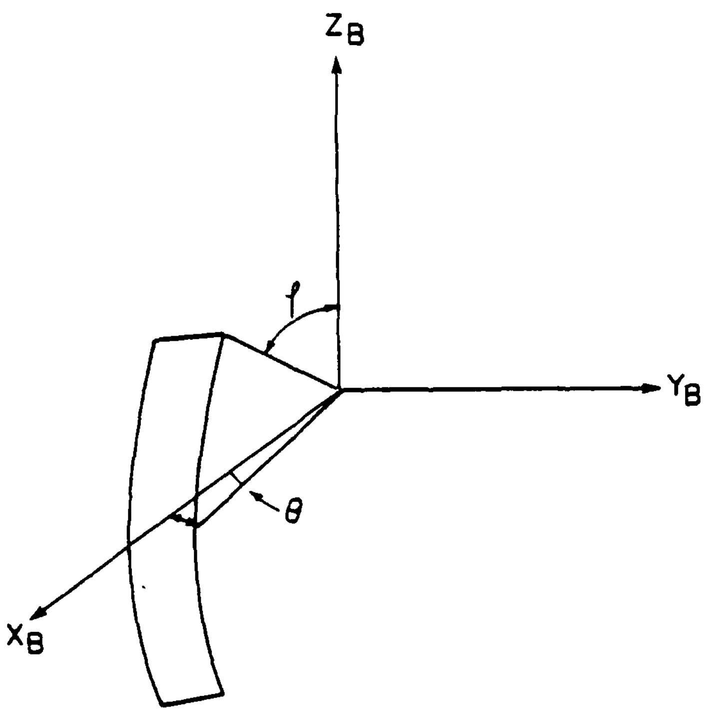

FIGURE 14 - ANGULAR DEFINITION OF A BARRIER (BSR SYSTEM)

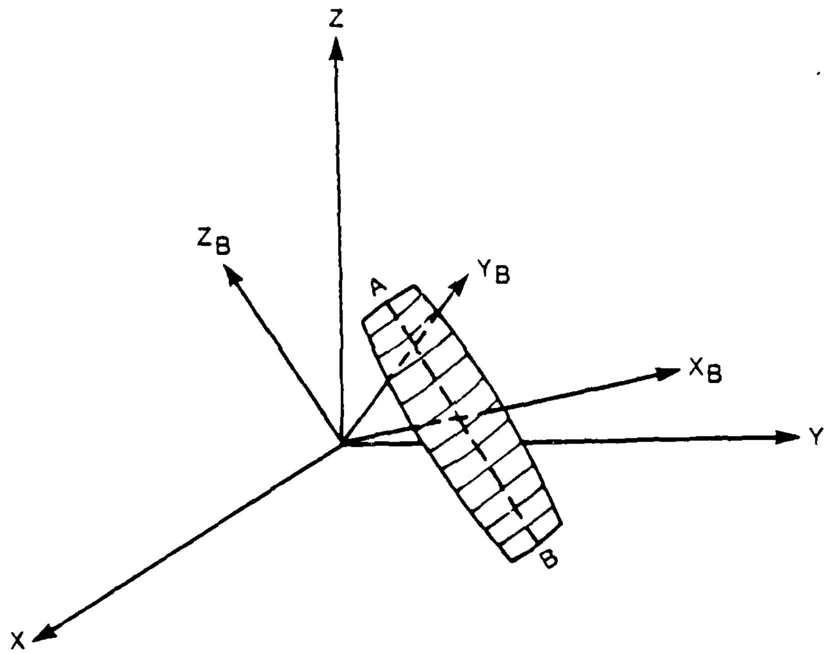

FIGURE 13 - SYSTEM OF COORDINATES ASSOCIATED WITH A BARRIER (BRR)

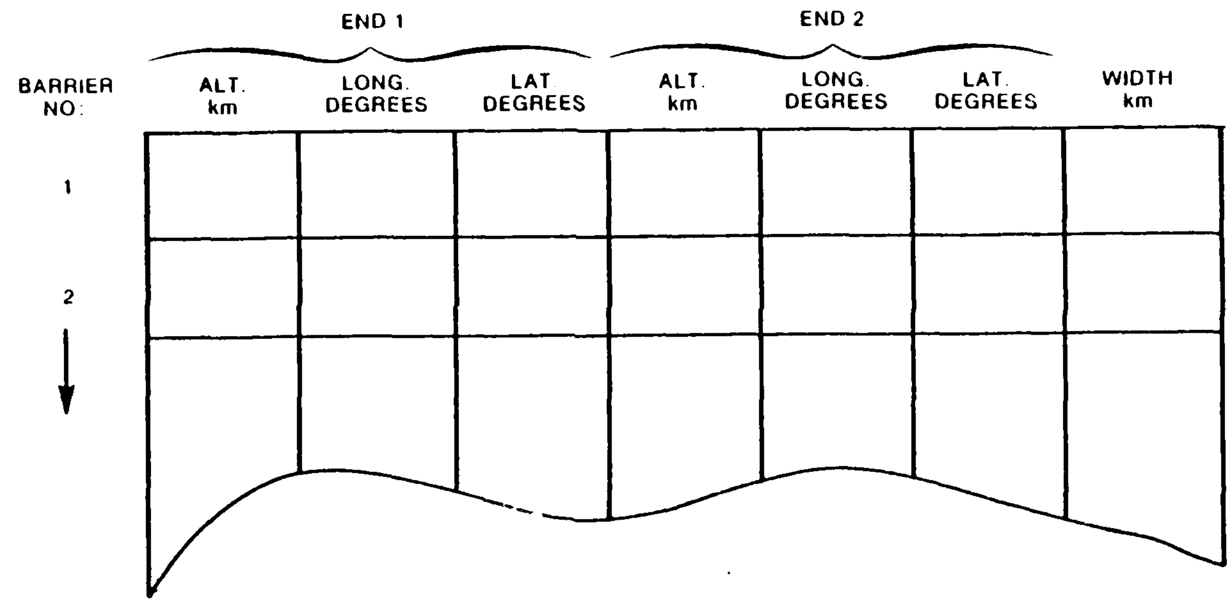

FIGURE 12 - FORMAT OF THE ARRAY BARRIER

parameters contained in RADPAR and SYSAT. More details on the scanning process may be found in the next section.

The parameters that define the location of the barrier are stored in an array called BARRIER (see Figure 12). A barrier is set up by the location of the two ends of its spine, the arc of great circle AB (see Figure 13). The width defined in the array BARRIER is the width at both ends of the barrier. The X_B axis should be going through the center of the cord AB. If we now consider a spherical system of coordinate (BSR) with the angle P measured from the Z_B axis (see figure 14), the barrier will be defined by

$$- L_\Theta < \theta < L_\Theta \qquad (10)$$

$$L_\phi < (\frac{\pi}{2} - \phi) < L_\phi$$

where L_Θ and L_ϕ are θ and ϕ half-angles subtended by the barrier from the origin of the system of coordinates. Then, a program calculates the location of the four corners of the barrier. The boundaries of the barrier are the areas of great circle joining the four corners. To check if a scan area or a coverage area has some overlap with a barrier, one has only to express the coordinates of the borders of that area in the BSR system and to check if some of them are within the limits of the barrier.

It is convenient to define two systems of coordinates associated with the barrier. These systems will be used to determine if some part of the scan lines or of the coverage areas of the radars are within the barrier. The first system (BRR) is a rectangular system centered on the Earth. The Z axis (Z_B on Figure 13) is parallel to the line AB joining the extremities A and B of the arc AB that is the spine of the barrier. The X axis (X_B on Figure 13) passes through the middle of the line AB. The second (BSR) system (see Fig. 14) is a spherical system derived from the BRR system with the angle ϕ measured from the Z axis.

SCANNING PROCESS

The size of the scan line generated by a radar depends on the parameters of the radar. They are stored in an array called RADPAR whose format is illustrated on Figure 15. The width of the beam is defined as the 50% point. The angular width of the beam is defined from the antenna size through the equation:

$$\Theta = \frac{1.2\lambda}{\pi \times S} \qquad (11)$$

where

λ = wavelength of the radar
s = size of the antenna.

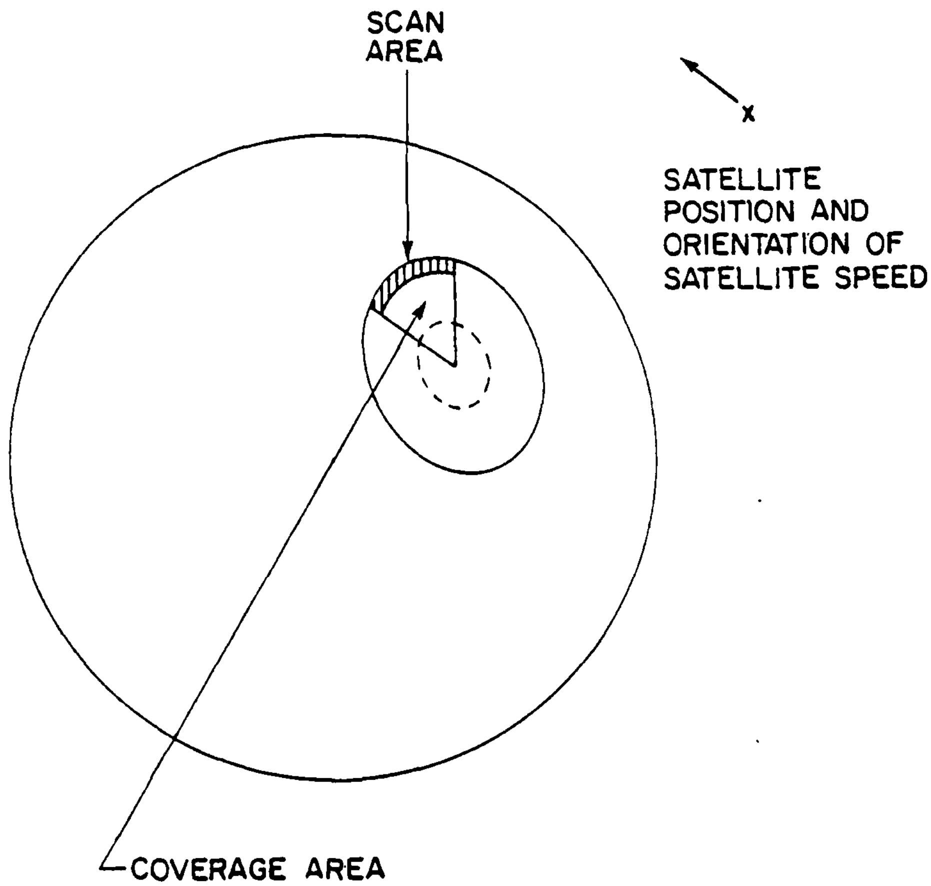

FIGURE 11 - SCAN AREA FOR THE MECHANICAL COVERAGE

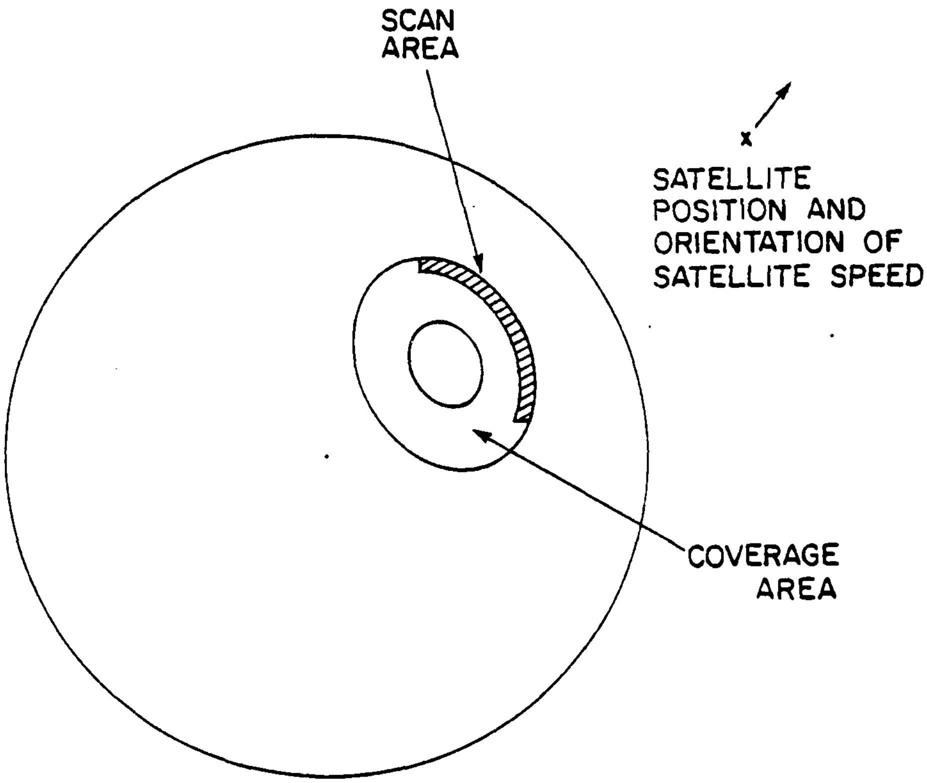

FIGURE 10 - SCAN AREA FOR THE ELECTRONIC COVERAGE

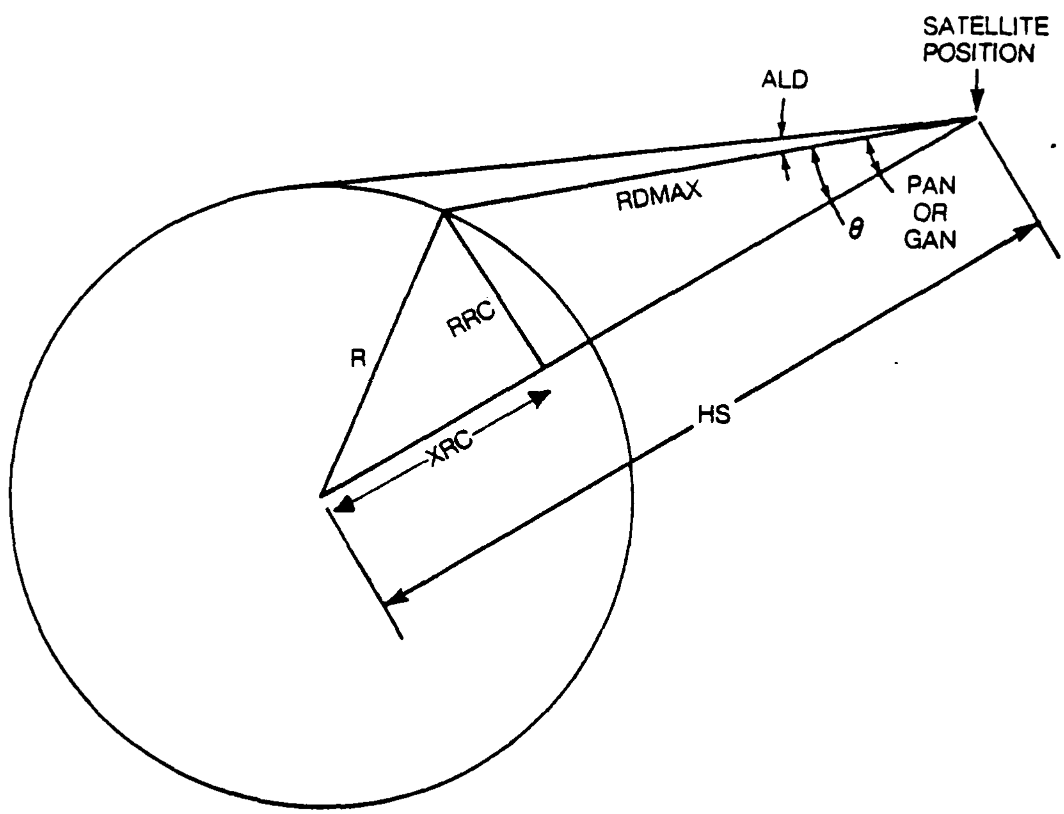

FIGURE 9 - GEOMETRICAL PARAMETERS INVOLVED IN THE CALCULATION OF THE COVERAGE AREA

The geometry of the situation is illustrated in Figure 9. ALD is the grazing angle stored in SYSAT. A few useful relationships between the parameters involved are listed here. There is no approximation in these equations and the Earth curvature has been taken into account. The Earth is assumed to be spherical with a radius R.

$$\Theta = \sin^{-1}\left(\frac{R}{HS} \times \sin\left(\frac{\pi}{2} + ALD\right)\right) \qquad (4)$$

$$BRC = HS \times tg\ \Theta \qquad (5)$$

$$MRC = tg\Theta \qquad (6)$$

$$XRC = \frac{(-BRC \times MRC) + \sqrt{R^2 + (MRC \times R)^2 - BRC^2}}{1 + MRC^2} \qquad (7)$$

$$RPC = (MRC \times XRC) + BRC \qquad (8)$$

$$RDMAX = \sqrt{(HS - XRC)^2 + RRC^2} \qquad (9)$$

The angle Θ can be either the angle associated with the outer border of the coverage area or the angle associated with the inner border of the coverage area. The parameter RDMAX is used in the program that checks if there is an airplane within the coverage area. RDMAX is the maximum distance from the satellite where you may find an airplane within the coverage area (on the same side of the Earth as the satellite).

SYSTEM OF BARRIERS

Out of many SBRS strategies, we choose to simulate the one in which the radars on the satellites operate only when the scan area overlaps some predefined areas called "barriers". Most of the time, in our simulation, the radars are in "search mode". It means that the scan areas of the radars are calculated at fixed intervals and a check is made to know if there is an overlap with the barrier. If a new overlap is found, the program will run a routine to find the time of the first contact with the barrier and from then, the radar will operate in scan mode as long as its scan area is not out of the barrier. The scan area of the electronic and mechanically scanned radar are shown on Figures 10 and 11 respectively. The width, the length and the location of the scan areas are calculated from the

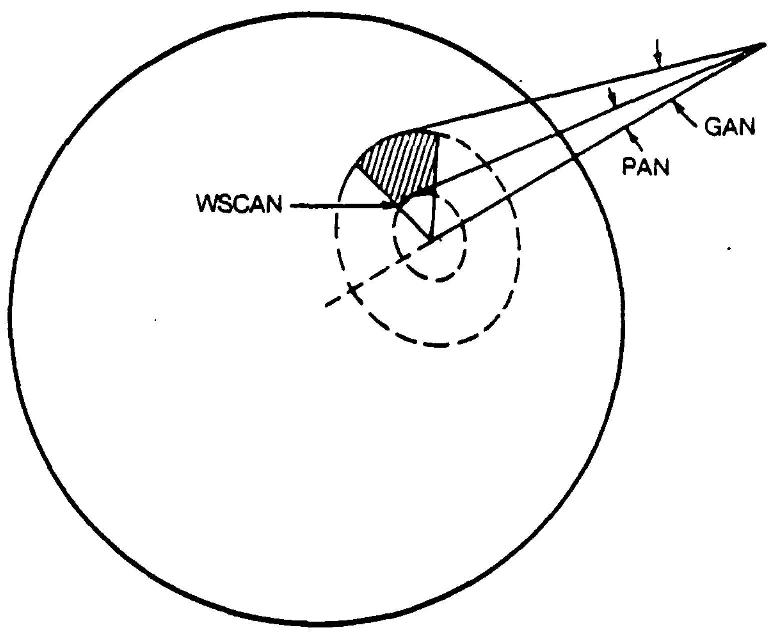

FIGURE 8 - MECHANICAL COVERAGE

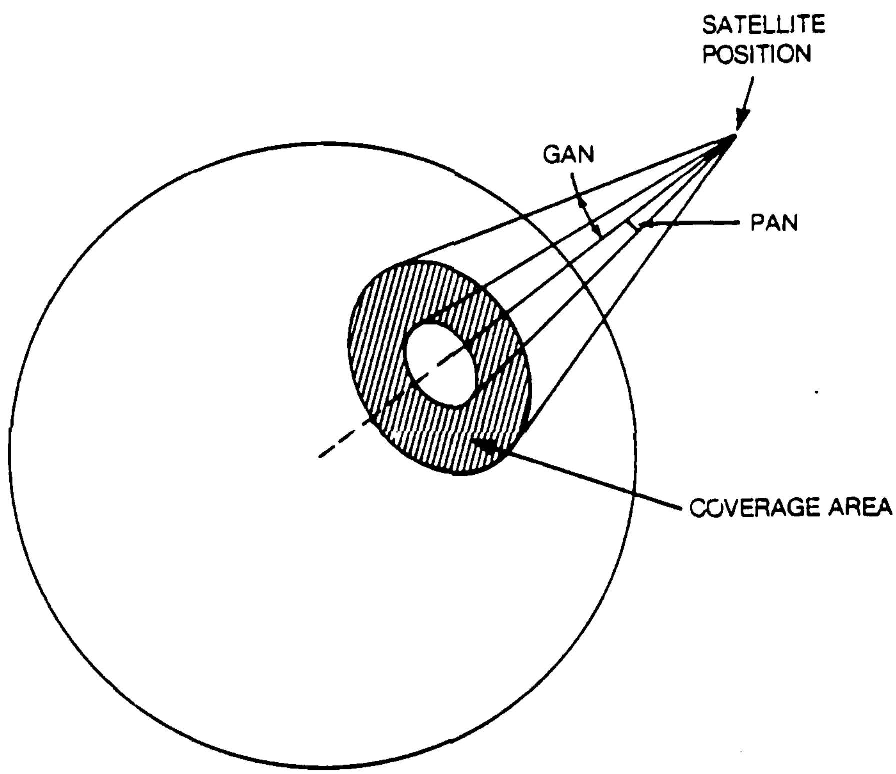

FIGURE 7 - ELECTRONIC COVERAGE

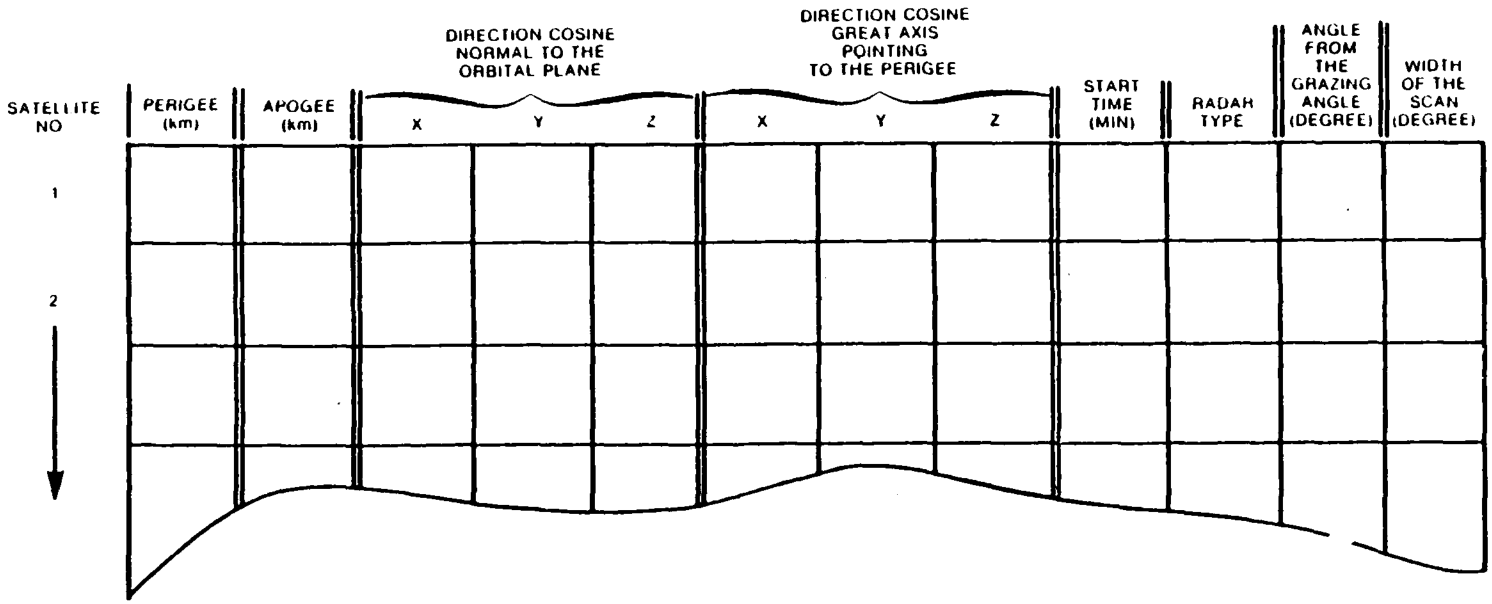

FIGURE 6 - FORMAT OF THE ARRAY SYSAT

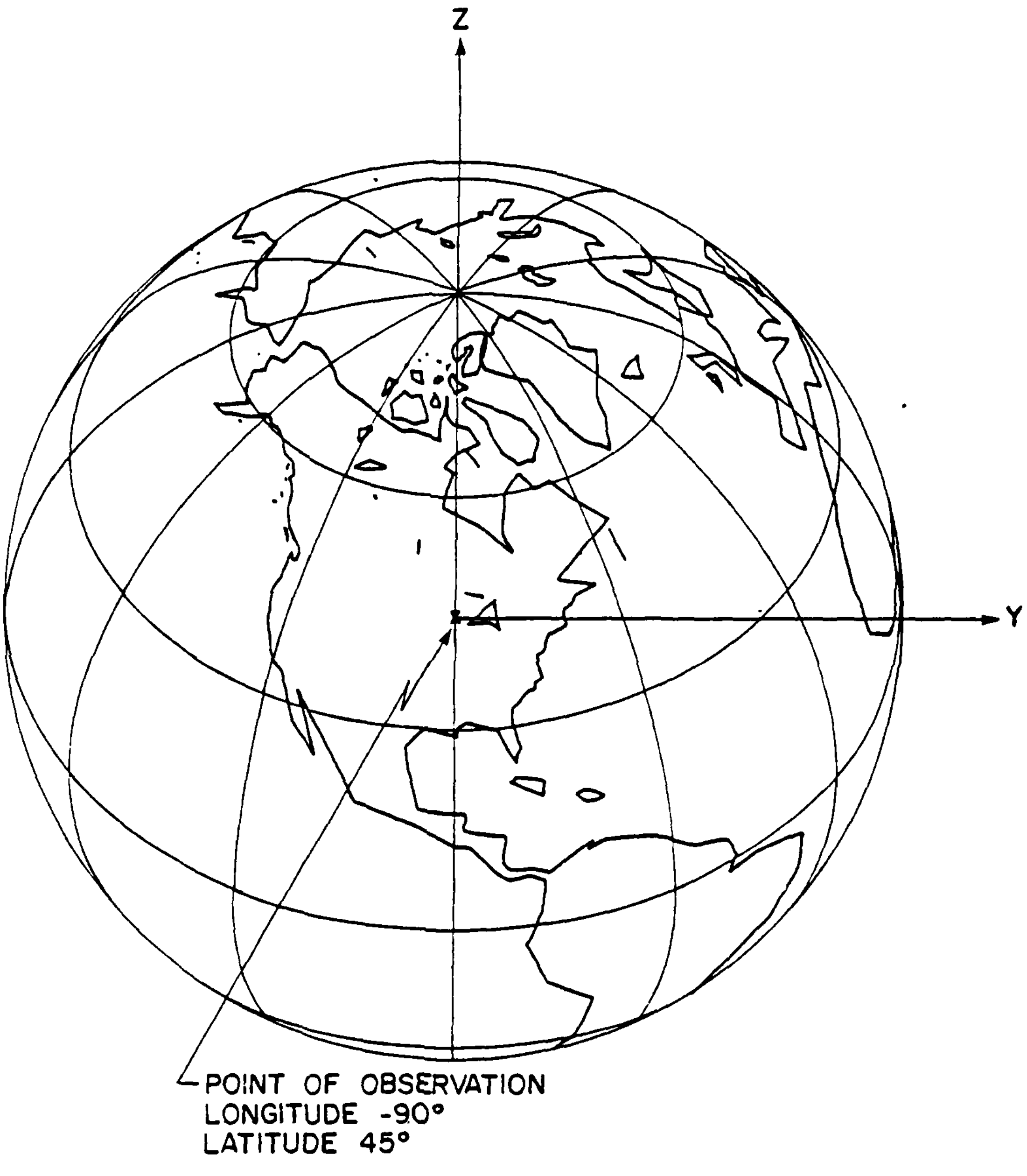

FIGURE 5 - SYSTEM OF COORDINATE TOPOCENTRIC TO THE POINT OF OBSERVATION (ORR)

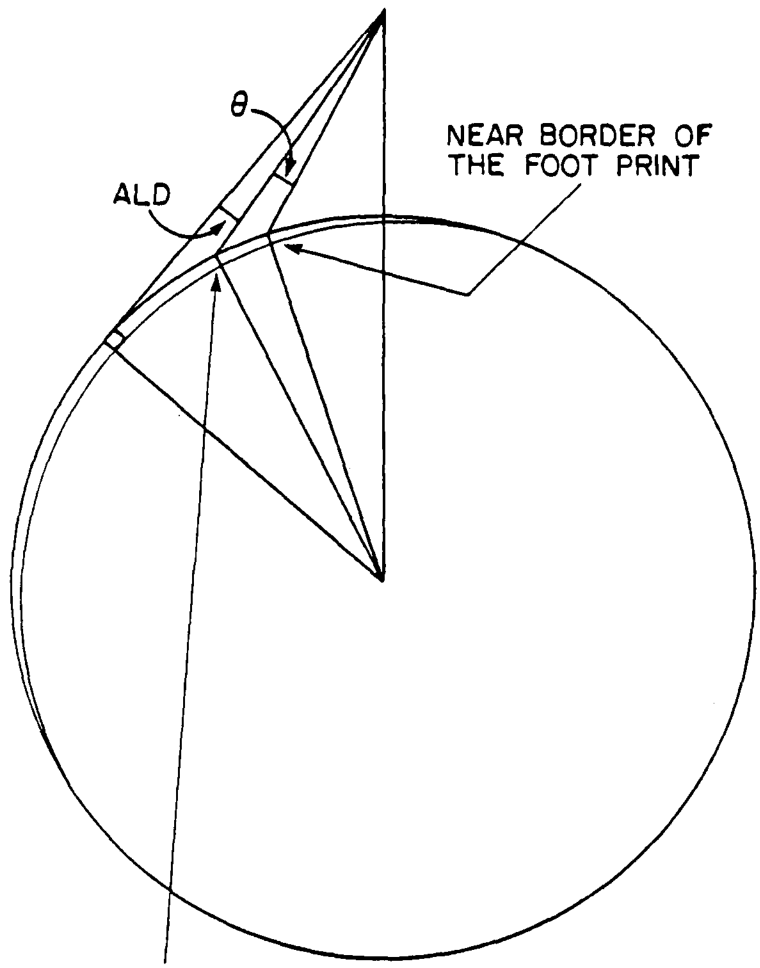

FIGURE 16 - GEOMETRY OF THE FAR AND NEAR LIMITS OF THE FOOTPRINT

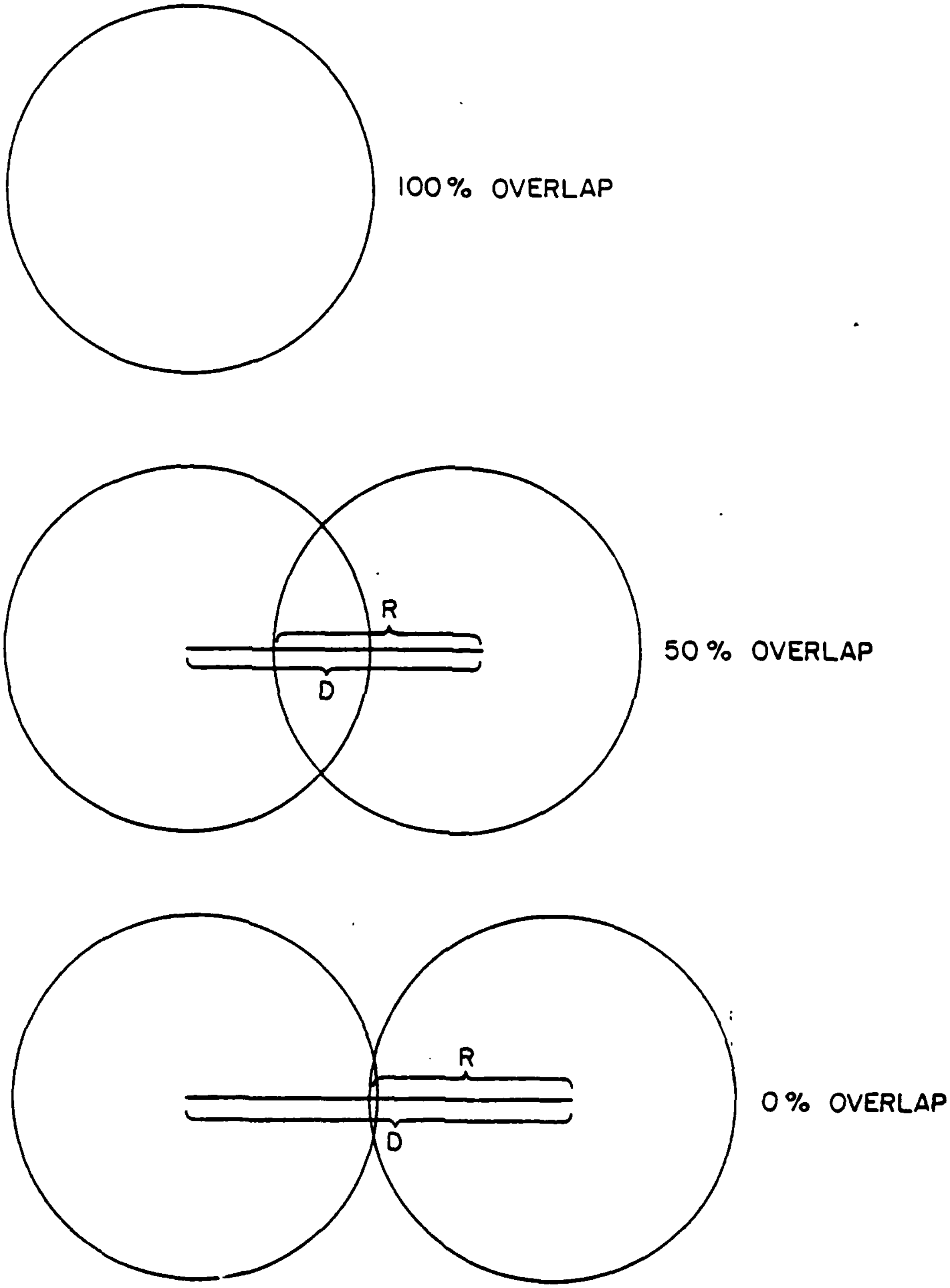

FIGURE 17 - ILLUSTRATION OF PERCENTAGE OF OVERLAP

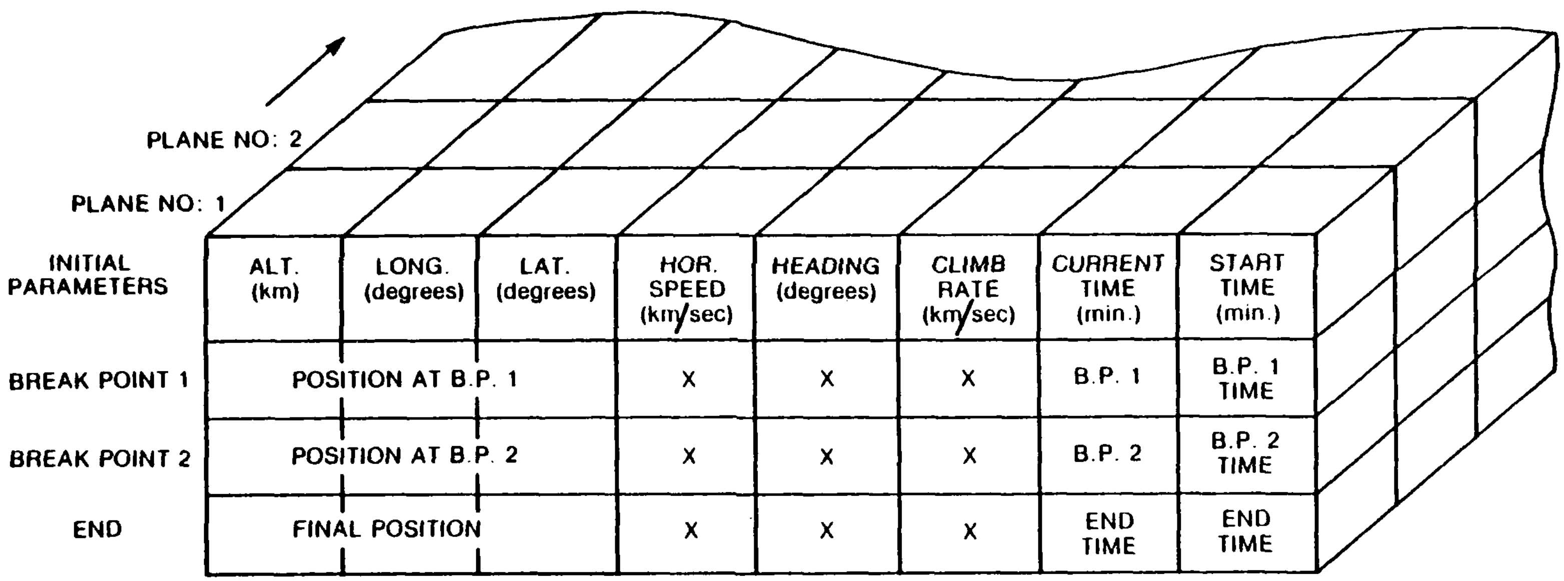

FIGURE 18 - FORMAT OF THE FP ARRAY

PLANE NO: 2

PLANE NO: 1

	ALT. (km)	LONG. (degrees)	LAT. (degrees)	HOR. SPEED (km/sec)	HEADING (degrees)	CLIMB RATE (km/sec)	CURRENT TIME (min.)	START TIME (min.)
INITIAL PARAMETERS								
BREAK POINT 1	—	—	—	X	X	X	0	B.P. 1 TIME
BREAK POINT 2	—	—	—	X	X	X	0	B.P. 2 TIME
END	—	—	—	X	X	X	0	END TIME

FIGURE 19 - REPARTITION OF THE DATA IN FP BEFORE AN EXECUTION OF A FLIGHT PLAN

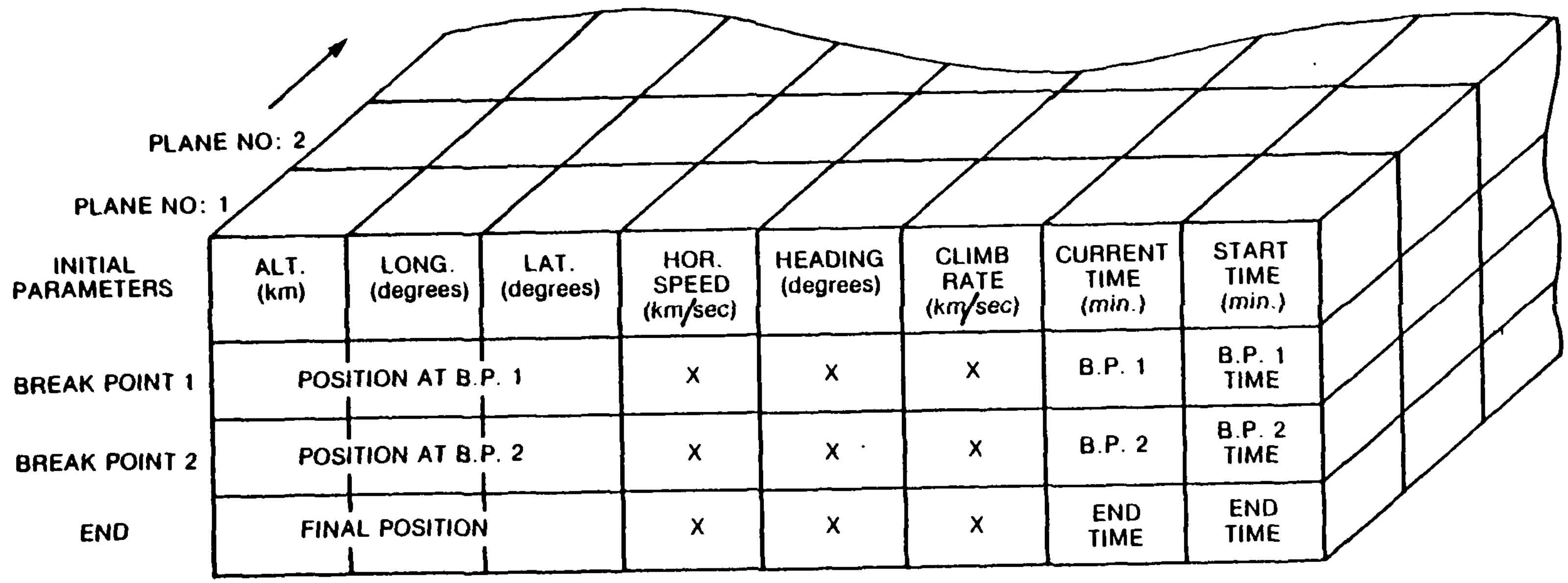

FIGURE 20 - REPARTITION OF THE DATA IN FP AFTER AN EXECUTION OF A FLIGHT PLAN

This particular approach to the handling of the data related to the movement of the airplanes has first been proposed and implemented by Dr. B. Bridgewater. We adapted the concept to the particular needs of our simulation.

It is to be noted that the system can operate only for positive time progression. The program will give erroneous data if the time is run in reverse.

The position of the airplane is always calculated from the preceeding breakpoint or from the start point. The airplanes are assumed to move on great-circles. The new position of the airplane is first calculated in a system of coordinate (PRC) centered on the Earth with the Z axis perpendicular to the plane containing the great-circle in which the airplane is moving. The X axis passes through the initial position of the airplane (see Figure 21). The angular movement of the airplane and its new position are calculated. The position of the airplane is then calculated in a system centered on the Earth with a topocentric orientation of its axis by using a proper matrix of rotation MRX (calculated by the program VALMFP). The position of the airplane is then calculated for display, in the system ORR by using the matrix of rotation MRFO.

RELATIVE SPEEDS

The first step to establish the signal to noise ratio associated with a particular satellite-target-Earth geometry consists of calculating the relative speed of the satellite and the target and the relative speed of the satellite and the clutter from the Earth behind the target as seen by the satellite. We also build a routine that calculates the relative speed between the satellite and 11 sample points located on the scan line (see Fig. 22).

We first calculate the speed of the satellite and of the target in the ERF system by using the matrix of rotation already defined. The component of the vector joining the satellite and the target or the satellite and the sampling point on the scan line in the ERF system to get the line on which the speed will be projected. The satellite, the target and the Earth speed are then projected on the proper lines and the Doppler shifts are calculated.

ROTATION OF COORDINATES

In order to perform the many rotations of coordinates generated by the programs, we build a routine that generates the matrix of rotation needed to go from a X'Y'Z' system to a XYZ system and the reverse. The routine ANGROT works from the components of the X' and Z' vector in the XYZ system and calculates the matrix MR to go from the X'Y'Z' system to the XYZ and the inverse matrix IMR to go from the XYZ to the X'Y'Z' system (see Figure 23). One may be surprised by the complexity of ANGROT that contains more than 4000 different cases. One should realize by example, that each time for, X'Y'or Z'are in one of the planes XOY, XOZ and XOY, we get a degenerated situation that has to be handled in its own way. A close examination of the problem leads to the identification of a very large number of such situations. This is why a check procedure has been added to ANGROT. It

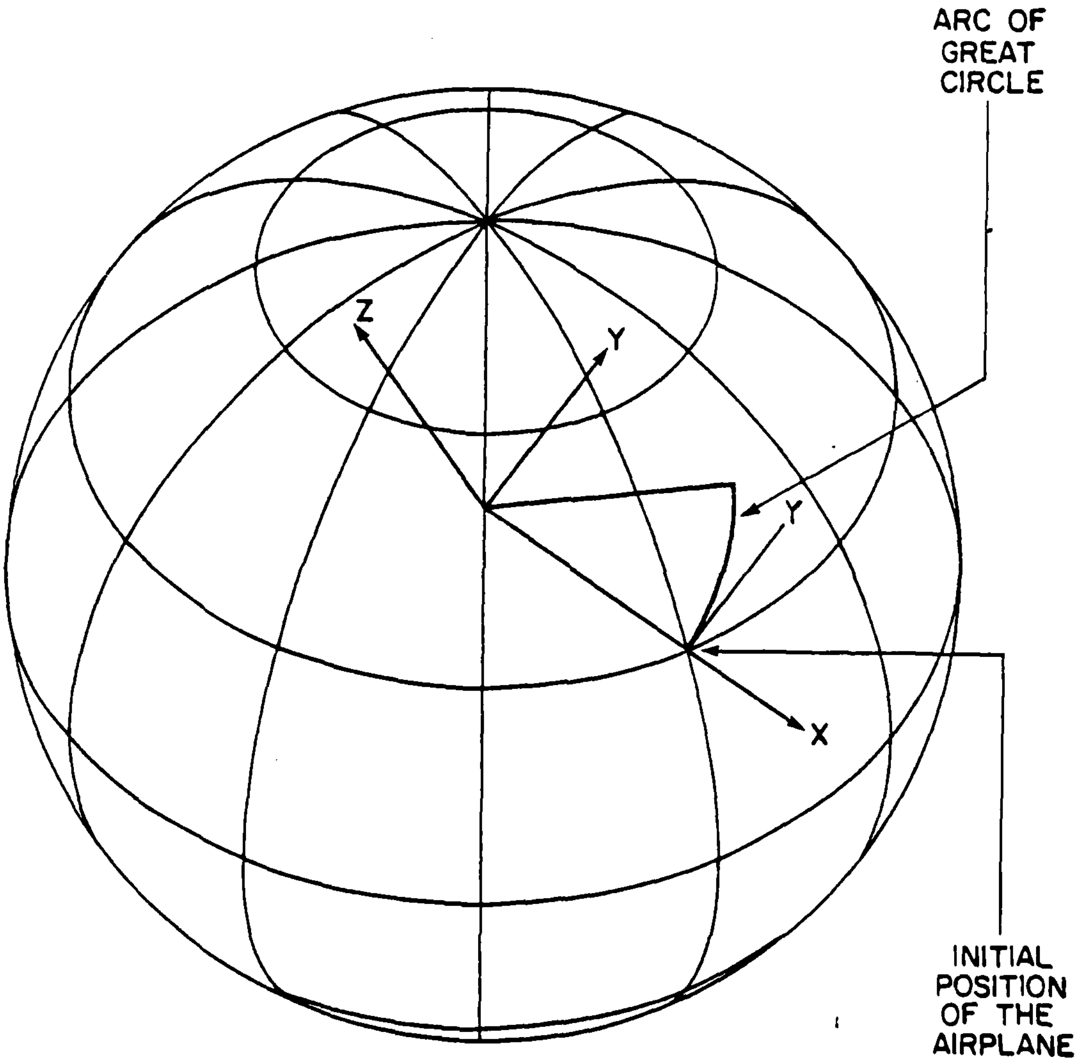

FIGURE 21 - SYSTEM OF COORDINATES RELATED TO THE AIRPLANE MOVEMENT

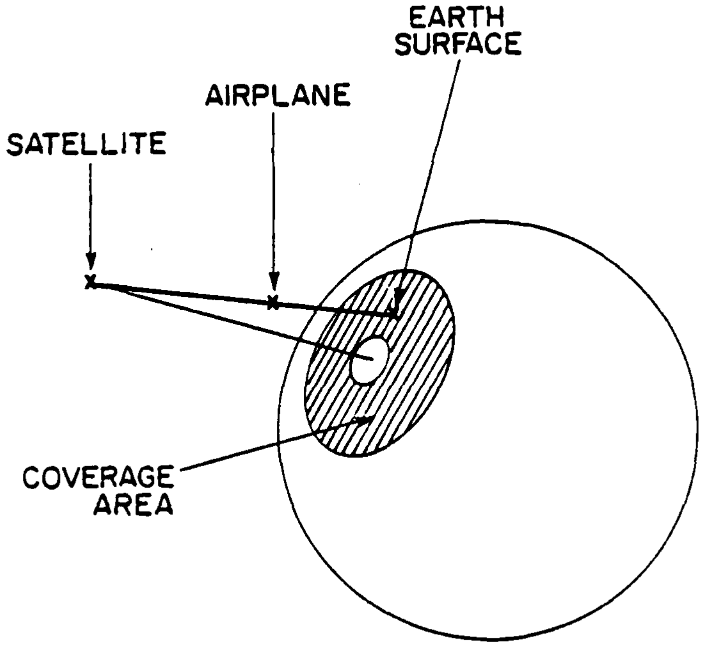

FIGURE 22 - ILLUSTRATION OF THE RELATIVE SPEED

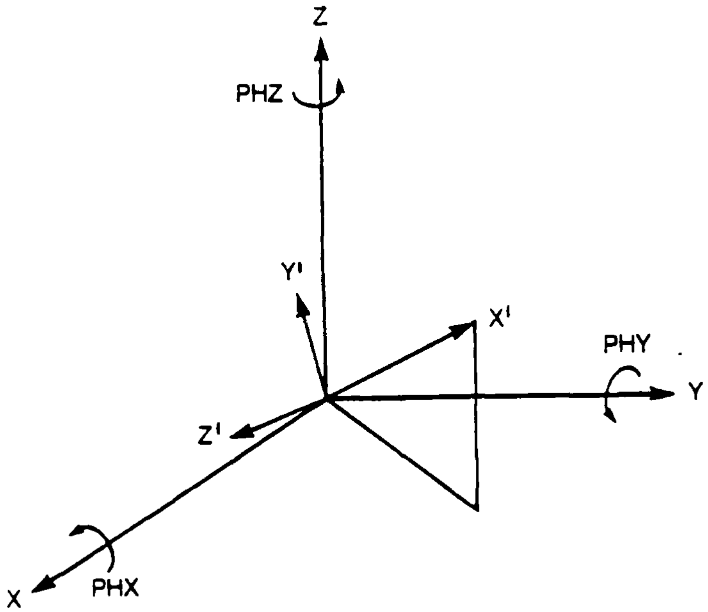

FIGURE 23 - ROTATION OF COORDINATES

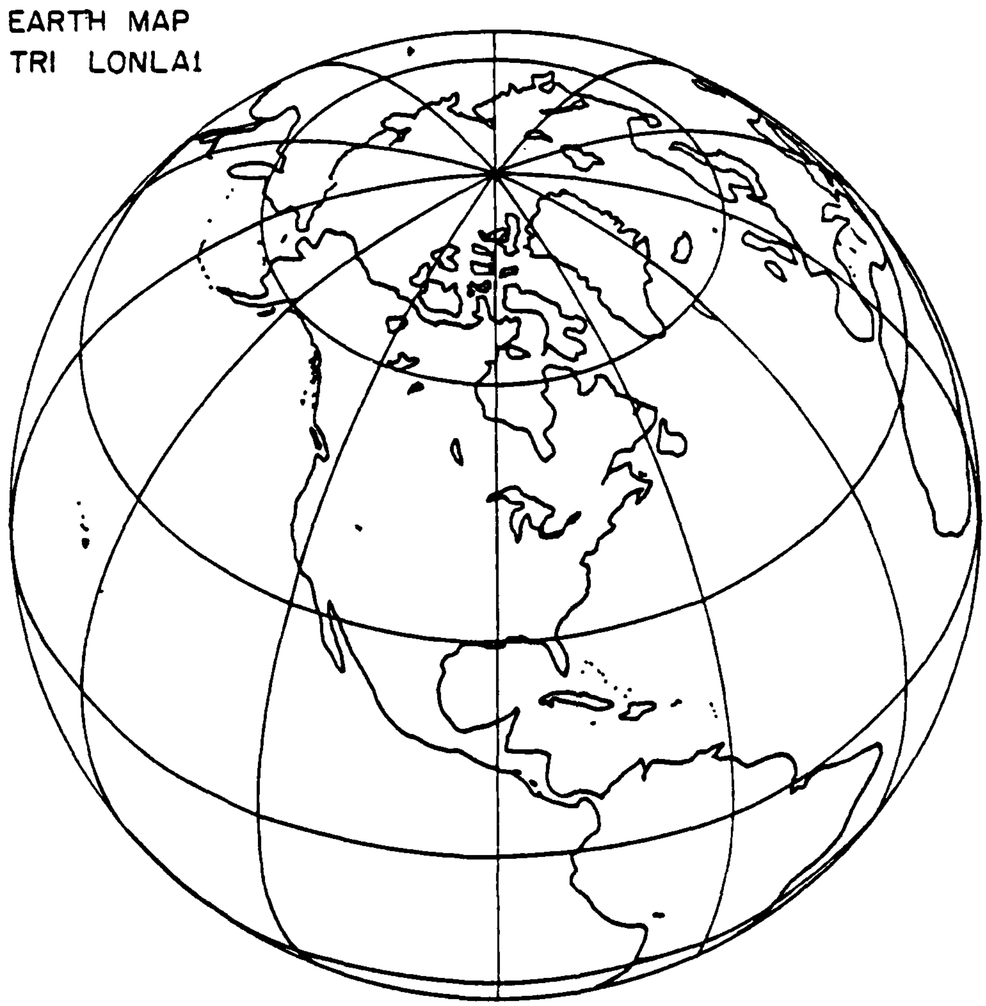

FIGURE 24 - MAP PRODUCED ON A TEKTRONIX SCREEN FROM THE DECISION SCIENCE ASSOCIATES INC. SOFTWARE

simply submits the X' axis, as defined in the XYZ system, to an IMR rotation. If the rotation is right, we will get as an answer the X' axis as defined in the X'Y'Z' system (1 0 0).

MAP PACKAGE

It is also possible to add to our display a nice outline of the boundaries of the continents, as seen from a point of observation (see Figure 24). The software package generating the data is written in Fortran language was gratuitously given to our group by Decision Science Associates Inc. This package is designed to calculate the projection of the boundaries of the Earth's continental features for various types of projection. Mr. B. Cook adapted these programs to our computer system and generated, in a proper format, a file containing the data associated with an orthonormal projection for an origin located at 90° of longitude West and 45° of latitude North. That file was read in APL, transferred to an APL workspace and processed.

We generated only the data necessary to produce a projection centered at 90° West and 45° North. Other maps may be generated at will by running the programs with other sets of parameters.

CONCLUSION

A brief description of the mathematical treatment involved in SBRS simulation has been presented. The principle of operation of the routine was also described. Special attention was paid to the statement of the condition of validity and of the approximation present in the various models.

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1) R. Deutsch, Orbital Dynamics of Space Vehicles, Prentice-Hall, 1963, p. 1 - 31.

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13 ABSTRACT

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